

Name: _____ Date: _____ Class: _____

ESSENTIAL QUESTION

How do the language and tools of functions — notation, domain/range, rate of change, composition, transformations, and inverses — work together to describe relationships between quantities?

Unit Overview

Unit 1 built the entire language of functions. A function assigns each input exactly one output (vertical line test), and its domain/range describe the possible inputs and outputs. The average rate of change measures how fast a function changes between two points — the slope of the secant line. Composition builds new functions by chaining f and g together, while transformations shift, stretch, and reflect a parent graph. An inverse function undoes another function, swapping domain and range.

$$\text{Avg. Rate of Change} = (f(b) - f(a)) / (b - a)$$

$$(f \circ g)(x) = f(g(x))$$

$$f(f^{-1}(x)) = x = f^{-1}(f(x))$$

Transform	$f(x-h)+k$ shifts h right, k up; $a \cdot f(x)$ stretches by $ a $; $f(-x)/-f(x)$ reflect
Domain	Roots: radicand ≥ 0 . Fractions: denominator $\neq 0$
Inverse	Exists (as a function) only where f is one-to-one — horizontal line test

Quick Reference

- Vertical line test: a graph is a function if every vertical line crosses it at most once.
- Horizontal line test: a function has an inverse function only if every horizontal line crosses it at most once.
- To find an inverse algebraically: swap x and y , solve for y , then verify with composition.

Worked Examples**EXAMPLE 1**

Evaluate $f(x) = 2x^2 - 3x + 1$ at $f(-2)$ and $f(a+1)$.

- $f(-2) = 2(4) - 3(-2) + 1 = 8 + 6 + 1 = 15$
- $f(a+1) = 2(a+1)^2 - 3(a+1) + 1 = 2a^2 + 4a + 2 - 3a - 3 + 1$
- Combine like terms: $f(a+1) = 2a^2 + a$

Answer: $f(-2) = 15$; $f(a+1) = 2a^2 + a$

EXAMPLE 2

Find the domain and range of $f(x) = \sqrt{x-3} + 2$.

- Domain: the radicand must be $\geq 0 \rightarrow x-3 \geq 0 \rightarrow x \geq 3$
- Range: $\sqrt{x-3} \geq 0$, so $f(x) = \sqrt{x-3} + 2 \geq 2$

Answer: Domain: $[3, \infty)$; Range: $[2, \infty)$

EXAMPLE 3

Find the average rate of change of $f(x) = x^2 - 4x$ on $[1, 5]$.

- $f(1) = 1 - 4 = -3$. $f(5) = 25 - 20 = 5$
- Avg. Rate of Change = $(f(5) - f(1)) / (5 - 1) = (5 - (-3)) / 4 = 8/4$

Answer: Average rate of change = 2

EXAMPLE 4

Let $f(x) = 2x+1$ and $g(x) = x^2-3$. Find $(f \circ g)(x)$ and $(g \circ f)(x)$.

- $(f \circ g)(x) = f(g(x)) = 2(x^2-3)+1 = 2x^2 - 5$
- $(g \circ f)(x) = g(f(x)) = (2x+1)^2 - 3 = 4x^2 + 4x + 1 - 3 = 4x^2 + 4x - 2$

Answer: $(f \circ g)(x) = 2x^2 - 5$; $(g \circ f)(x) = 4x^2 + 4x - 2$

EXAMPLE 5

Find the inverse of $f(x) = (x+5)/3$, and verify.

- Swap x and y : $x = (y+5)/3 \rightarrow 3x = y+5 \rightarrow y = 3x - 5$
- Verify: $f(f^{-1}(x)) = f(3x-5) = (3x-5+5)/3 = 3x/3 = x \checkmark$

Answer: $f^{-1}(x) = 3x - 5$

Guided Practice

Work through each problem below. Use the hint if you get stuck — write your final answer on the last line.

- 1** Evaluate $f(x) = -x^2 + 5x - 2$ at $f(3)$ and $f(-1)$.

Hint: Substitute carefully and watch your signs — square before multiplying.

- 2** Find the domain and range of $f(x) = 1/(x+4) - 1$.

Hint: The denominator can't be 0. The horizontal asymptote tells you the excluded range value.

3 Find the average rate of change of $f(x) = x^3$ on $[-1, 2]$.

Hint: Compute $f(-1)$ and $f(2)$ first, then apply the average rate of change formula.

4 Given $f(x) = \sqrt{x}$ and $g(x) = x+4$, find $(f \circ g)(x)$ and its domain.

Hint: Substitute $g(x)$ into f , then require the radicand to be ≥ 0 .

5 Find the inverse of $f(x) = 2x^3 - 1$.

Hint: Swap x and y , isolate y^3 , then take a cube root of both sides.

Key Vocabulary

Function	A relation where every input (x) corresponds to exactly one output (y).
Domain / Range	The set of all possible input values (domain) and output values (range) of a function.
Average Rate of Change	The slope of the secant line between two points: $(f(b)-f(a))/(b-a)$.
Composition of Functions	Combining two functions so the output of one becomes the input of the other: $(f \circ g)(x) = f(g(x))$.
Transformation	A shift, stretch, compression, or reflection applied to a parent function's graph.
Piecewise Function	A function defined by different rules on different intervals of its domain.
Inverse Function	A function that reverses another function's mapping; exists only where the original is one-to-one.
One-to-One	A function where each output corresponds to exactly one input — passes the horizontal line test.

Check Your Understanding

Circle the letter of the correct answer for each question.

1 Which set of ordered pairs is NOT a function?

Multiple Choice

A $\{(1,2), (2,3), (3,4)\}$

B $\{(1,2), (1,3), (2,4)\}$

C $\{(0,0), (1,1), (2,2)\}$

D $\{(-1,5), (0,5), (1,5)\}$

2 What is the domain of $f(x) = \sqrt{5-x}$?

Multiple Choice

A $(-\infty, 5]$ **B** $[5, \infty)$ **C** $(-\infty, 5)$ **D** All real numbers**3** If $f(x)=3x-2$ and $g(x)=x+1$, find $(f \circ g)(2)$.

Multiple Choice

A 4**B** 5**C** 7**D** 9**4** Which transformation shifts $f(x)$ left 3 and down 2?

Multiple Choice

A $f(x+3) - 2$ **B** $f(x-3) + 2$ **C** $f(x+3) + 2$ **D** $f(x-3) - 2$ **5** Which function is the inverse of $f(x) = (x-4)/2$?

Multiple Choice

A $f^{-1}(x) = 2x+4$ **B** $f^{-1}(x) = 2x-4$ **C** $f^{-1}(x) = (x+4)/2$ **D** $f^{-1}(x) = 2x+8$

Common Mistakes to Avoid

✗ Confusing $f(x+2)$ with $f(x)+2$ — assuming both shift the graph the same way.

✓ $f(x+2)$ shifts LEFT 2 (inside affects x , opposite sign); $f(x)+2$ shifts UP 2 (outside affects y , same sign).

✗ Assuming every equation or graph is a function without applying the vertical line test.

✓ Check that every input has exactly one output — for graphs, no vertical line crosses more than once.

✗ Forgetting to check the domain of a composite function.

✓ The domain of $(f \circ g)(x)$ must satisfy restrictions from BOTH g 's domain and f 's domain applied to $g(x)$.

✗ Finding an inverse by swapping x and y but forgetting to solve for y , or ignoring one-to-one restrictions.

✓ Always finish solving for y , and restrict the domain first if the original function isn't one-to-one.

Math Tips

- ★ Domain of a root: set the radicand ≥ 0 . Domain of a fraction: set the denominator $\neq 0$.
- ★ Average rate of change is just the slope of the line connecting two points on the graph.
- ★ Inside-the-function changes (h) move horizontally OPPOSITE the sign; outside changes (k) move vertically WITH the sign.
- ★ A function has an inverse function only where it's one-to-one — check with the horizontal line test.
- ★ To verify an inverse, confirm BOTH $f(f^{-1}(x))=x$ AND $f^{-1}(f(x))=x$.